Learning about the chiral structure of the proton from the hyperfine splitting*

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The complete analytical $O(m_l^3\alpha^5/m_p^2\times(\log m_\pi,\log\Delta,\log m_l))$ contribution to the Hyperfine splitting is given. It can explain about 2/3 of the difference between experiment and the pure QED prediction when setting the renormalization scale at the ρ mass. The suppression of the polarizability piece with respect the Zemach one seems to be, to a large extent, a numerical accident. We give an estimate of the matching coefficient of the spin-dependent proton-lepton operator in heavy baryon effective theory.

High precision measurements in atomic physics provide with a unique place to determinate some hadronic parameters related with the proton elastic and inelastic electromagnetic form factors, like the proton radius and magnetic moment, polarization effects, etc.... One complication in this program comes from the fact that widely separated scales are involved in these physical processes. Therefore, it becomes important to relate the physics at these disparate scales in a model independent way. Effective field theories (EFT's) are a natural approach to this problem. In particular, we need an EFT at atomic scales. We will use potential NRQED [1]. Its effective Lagrangian for QED weakly bound systems at $O(m\alpha^5)$ can be found in Ref. [2]. It basically reduces to a Schrödinger equation interacting with ultrasoft photons. Here we are concerned with the hadronic contributions to this EFT (see [3]). They are encoded in the matching coefficients, i.e. in the potentials. Moreover, we will focus on the logarithmically enhanced hadronic effects to the hyperfine splitting, which, at the order of interest, only appear in the delta potential[2]:

$$\delta V = 2\frac{c_{4,NR}^{pl}}{m_p^2}\mathbf{S}^2\delta^{(3)}(\mathbf{r})\,. \tag{2}$$

The point here is to obtain the coefficient $c_{4,NR}^{pl}$ from QCD in a controlled way. In practice, what one can do is to compute its chiral structure due to energies of $O(m_\pi)$ and to parameterize the effects due to energies of $O(m_\rho)$ with matching coefficients inherited from Heavy Baryon Effective Theory [4].

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[2]We will not consider here the hadronic effects inherited from the anomalous magnetic moment of the proton, μ_p, due to the tree level potential:

$$\delta V = \frac{4\pi\alpha(1+\mu_p)}{3m_pm_l}\mathbf{S}^2\delta^{(3)}(\mathbf{r})\,, \tag{1}$$

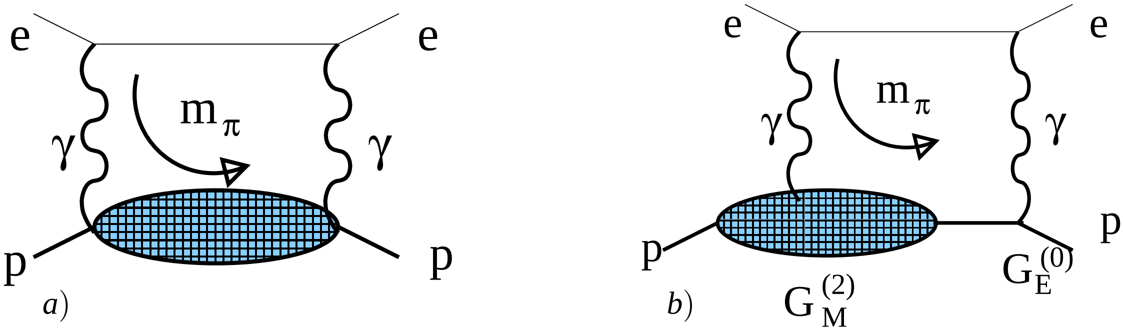

Figure 1. *Figure a) corresponds to Eq. (3). Figure b) corresponds to Eq. (6).*

The first non-vanishing contribution to $c_{4,NR}$ appears at $O(\alpha^2)$. Its leading order expression reads (an infrared cutoff larger than $m_l\alpha$ is understood and the expression for the integrand should be generalized for an eventual full computation in D dimensions)

$$c_{4,NR}^{pl} = -\frac{ig^4}{3}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2}\frac{1}{k^4-4m_l^2k_0^2}\left\{A_1(k_0,k^2)(k_0^2+2k^2)+3k^2\frac{k_0}{m_p}A_2(k_0,k^2)\right\}, \tag{3}$$

consistent with the expressions obtained long ago as in Ref. [5]. It is symbolically depicted in Fig. 1a). The bubble is meant to represent the hadronic structure of the proton, which, at the order of interest, is represented by the forward virtual-photon Compton tensor (needed at $O(1/F_0^2)$ [6,3] for the spin-dependent terms),

$$T^{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle p,s|TJ^\mu(x)J^\nu(0)|p,s\rangle\,, \tag{4}$$

which has the following structure ($\rho = q\cdot p/m$):

$$\begin{aligned} T^{\mu\nu} = \quad & \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)S_1(\rho,q^2)+\frac{1}{m_p^2}\left(p^\mu-\frac{m_p\rho}{q^2}q^\mu\right)\left(p^\nu-\frac{m_p\rho}{q^2}q^\nu\right)S_2(\rho,q^2) \\ & -\frac{i}{m_p}\,\epsilon^{\mu\nu\rho\sigma}q_\rho s_\sigma A_1(\rho,q^2)-\frac{i}{m_p^3}\,\epsilon^{\mu\nu\rho\sigma}q_\rho\left((m_p\rho)s_\sigma-(q\cdot s)p_\sigma\right)A_2(\rho,q^2). \end{aligned} \tag{5}$$

In many cases only the so called Zemach correction [7] is considered. It corresponds to the Born approximation of the above expressions (see Fig. 1b)). It reads (for the definitions see [3])

$$\delta c_{4,Zemach}^{pl} = (4\pi\alpha)^2 m_p\frac{2}{3}\int\frac{d^{D-1}k}{(2\pi)^{D-1}}\frac{1}{\mathbf{k}^4}G_E^{(0)}G_M^{(2)}\,. \tag{6}$$

It turns out to be the dominant contribution even if, formally, the other pieces are equally important from the power counting point of view.

The total sum in the SU(2) case reads (including point-like, Zemach and polarizability effects) [3][3]

$$\begin{aligned} \delta c_{4,NR}^{pl} \;\simeq\; & \left(1-\frac{\mu_p^2}{4}\right)\alpha^2\ln\frac{m_l^2}{\nu^2}+\frac{b_{1,F}^2}{18}\alpha^2\ln\frac{\Delta^2}{\nu^2}+\frac{m_p^2}{(4\pi F_0)^2}\alpha^2\frac{2}{3}\left(\frac{2}{3}+\frac{7}{2\pi^2}\right)\pi^2 g_A^2\ln\frac{m_\pi^2}{\nu^2} \\ & +\frac{m_p^2}{(4\pi F_0)^2}\alpha^2\frac{8}{27}\left(\frac{5}{3}-\frac{7}{\pi^2}\right)\pi^2 g_{\pi N\Delta}^2\ln\frac{\Delta^2}{\nu^2}\,, \end{aligned} \tag{7}$$

since μ_p is known with very high precision from other sources.

[3]Note that a dipole parameterization of the form factors (see, for instance, [8,9]) would be unable to give the chiral logs, since they do not incorporate the correct non-analytical behavior in the momentum dictated by the chiral symmetry.

where we have used (the definition of C can be found in the Appendix of Ref. [3])

$$C = \frac{\pi^3}{12} - \frac{7}{8}\pi = -0.165037\,, \tag{8}$$

for the polarizability corrections (see [3]). We can see that there appears to be a numerical cancellation compared with the natural size of each term in C. This is consistent with the fact that, experimentally, polarizability corrections seem to be small [10]. It is also interesting to consider the large N_c limit of Eq. (7), it reads (we neglect $\ln(\Delta/m_\pi)$ terms for consistency)

$$\delta c^{pl}_{4,NR}(N_c \to \infty) \simeq \alpha^2 \ln\frac{m_l^2}{\nu^2} + \frac{m_p^2}{(4\pi F_0)^2}\alpha^2\pi^2 g_A^2 \ln\frac{m_\pi^2}{\nu^2}\,. \tag{9}$$

With Eq. (7), one can obtain the leading hadronic contribution to the hyperfine splitting. It reads (μ_{lp} is the reduced mass)

$$E_{\rm HF} = 4\frac{c^{pl}_{4,NR}}{m_p^2}\frac{1}{\pi}(\mu_{lp}\alpha)^3\,. \tag{10}$$

By fixing the scale $\nu = m_\rho$ we obtain the following number for the total sum in the SU(2) case:

$$E_{\rm HF,logarithms}(m_\rho) = -0.031\ {\rm MHz}\,. \tag{11}$$

Equation (11) accounts for approximately 2/3 of the difference between theory (pure QED) [8] and experiment [11]. What is left gives the expected size of the counterterm. Experimentally what we have is $c^{pl}_{4,NR} = -48\alpha^2$ and $c^{pl}_{4,R}(m_\rho) \simeq c^{p}_{4,R}(m_\rho) \simeq -16\alpha^2$. This last figure gives the expected size of the matching coefficient that appears in the heavy baryon effective Lagrangian:

$$\delta\mathscr{L}_{(N,\Delta)l} = \frac{1}{m_p^2}\sum_l c^{pl}_{4,R}\bar{N}_p\gamma^j\gamma_5 N_p\ \bar{l}\gamma_j\gamma_5 l\,. \tag{12}$$

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